On Closed Einstein-de Sitter Universes

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Abstract

We briefly summarize the idea of cosmological models with compact, flat spatial sections. It has been suggested that, because of the COBE satellite's maps of the microwave background, such models cannot be small in the sense of Ellis, and hence are no longer interesting. Here we use Lehoucq et al.'s method of cosmic crystallography to show that these models are physically meaningful even if the size of the spatial sections is of the same order of magnitude as the radius of the observational horizon.

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I. INTRODUCTION

Einstein-de Sitter cosmological model (EdS) belongs to the Friedmann-Lemaître-Robertson-Walker family, with null cosmological constant and matter density ratio $\Omega_0 = 1$. In the case of a pressureless energy-momentum tensor as the source in Einstein equation, which represents today's universe, EdS's metric is

$$ds^2 = c^2 dt^2 - (t/t_0)^{4/3} \left(dx^2 + dy^2 + dz^2 \right), \tag{1}$$

where t_0 is the present age of the universe in this model. Usually the spatial sections of EdS are taken to be the infinite Euclidean space E^3, which has a trivial global topology, i.e., it is a

simply connected space. But we may also consider closed (i.e., compact and without border) Euclidean manifolds M^3, which are related to E^3 through the isometry $M^3 \simeq E^3/\Gamma$, where Γ is a discrete group of isometries of E^3; they have a nontrivial global topology, with the fundamental group isomorphic to Γ. See Wolf [1] for a rigorous mathematical development, or Ellis[2] for a more graphical description (unfortunately the latter contains a mistake, which will be discussed below). For cosmology the interesting closed Euclidean manifolds (CEMs) are the six orientable ones, and here we use the labels $E1 - E6$ for them, as in Lachièze-Rey and Luminet [3], hereafter LaLu. These CEMs can be obtained by identification of pairs of faces of a cube ($E1 - E4$) or a rectangular prism with a regular hexagon as basis ($E5$, $E6$). Such a polyhedron is called a *Dirichlet domain,* or *fundamental polyhedron* (FP) for the manifold. The FP with identified face pairs represents comoving space, and is the real space where the sources are. The elements of Γ act upon the FP to generate its replicas in E^3, which is the locus of the repeated images of the sources in the FP. See, for example, LaLu's review.

Ellis and Schreiber [4], hereafter E&S, presented the idea that, with a universe not only closed but also small, we might have an alternative to the inflationary scenario as an explanation for the large scale uniformity of the distribution of matter in the universe. Their point is that, if the dimensions of the FP are of the order of a few hundred megaparsecs, then one would not need an actual homogeneous distribution in real space, the observed large scale uniformity being the result of repeated images of the sources in the FP.

Recently a number of authors - Sokolov [5], Stevens, Scott, and Silk [6], de Oliveira-Costa and Smoot [7], de Oliveira-Costa, Smoot, and Starobinsky [8], and others - have argued that the maps of the cosmic microwave background radiation produced by the NASA satellite Cosmic Background Explorer (COBE), put large lower limits on the size of the FP, should one adopt a 3-torus T^3, which is the simplest CEM ($E1$ in LaLu and here), as a model for cosmic space. Let Hubble's constant be $H_0 = 100h$ km s^{-1}Mpc^{-1}, where $0.4 \leq h \leq 1.0$, and $R_H = 2c/H_0 = 6000h^{-1}$ Mpc be the radius of the observational horizon. If we take a cube with edge of comoving length L as the FP, then the smallest of these limits known to

the authors is the one obtained in Ref. [5], $L \geq 0.7R_H$, and the largest is that of Ref. [7], $L \geq 1.2R_H$. Because the dimensions of the the FP turn out not to be small in E&S's sense, Refs. [6–8] imply that models with nontrivial topology are no longer interesting.

But these results have been obtained by harmonic analysis of COBE maps, tacitly assuming that each spot on the surface of last scattering (SLS) should be an image of a density fluctuation with diameter smaller than that of the FP. As Roukema [9] has argued, a fluctuation might cross the border of the FP, with the net result that its extension would be larger than the diameter of a single cell intersecting the SLS. For example, if we think of a fluctuation that "spirals" a number n of times around the 3-torus, then those lower limits for L would have to be divided by n and we might recover a small universe in the sense of E&S.

The above point deserves a more detailed study. Meanwhile, in this paper we adopt the large limits as valid, to argue that even then one can predict observable effects for such models, which of course are not small universes. The point is that the multiple connectedness of their topology has its own consequences, even if their size is a large fraction of the observable universe's volume. We use the method proposed by Lehoucq, Lachièze-Rey and Luminet [10], hereafter LeLaLu, based on a plot of distances between cosmic images. Because of the periodicities involved in CEMs, one obtains sharp peaks in this distribution, while the prediction for an infinite universe (or a finite one containing the *whole* observable region) presents a distribution without such peaks.

II. COSMIC CRYSTALLOGRAPHY

As discussed in Ref. [2], space sections of cosmological spacetimes are orientable, so the CEMs referred to in this paper are the orientable ones. There are six families of these, as classified by their global topology. Their rigorous mathematical study is found in [1]; Ellis [2] interpreted Wolf's esoteric expressions in colloquial terms, as summarized in E&S's Table 2. Unfortunately type $T = 4$ in this table, which we call $E4$, is described as resulting from the identification of opposite sides of a cube with "all pairs rotated by 180°." But, as pointed

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out by Bernui et al. [11], this prescription generates the projective space P^3, which does not admit a Euclidean metric. A correct pictorial description was obtained by Gomero [12], and here we use a modified version of his result: With a coordinate system (x, y, z) with origin at the FP's center and axes perpendicular to its faces, a set of face-pairing generators for group Γ in $E4 = E^3/\Gamma$ is $\{a, b, c\}$, with

$$a(x, y, z) = (x + L, \ -y, \ -z) \ ,$$
$$b(x, y, z) = (-x, \ z + L, \ y) \ , \qquad (2)$$
$$c(x, y, z) = (-x, \ z, \ y + L) \ ,$$

which satisfy the relations $a^{-1}ca^{-1}b = caba = b^{-2}c^2 = 1$.

As for the other five CEMs, they are described by their face pairings in LaLu, Table 17, as $E1 - E3$, $E5$, $E6$. (Their $E4$ is based on E&S's mistaken rule for $T = 4$.) The basic prediction of multiply connected universes is the formation of repeated images of cosmic sources. But the detection and recognition of multiple images is not an easy task; see LaLu, §11.2. Among the alternative proposals for discerning a nontrivial cosmic topology is the *cosmic crystallography* idea of LeLaLu, which we illustrate here by an example: Take an image in $E4$ at point $P = (x, y, z)$ and another at $bab^{-1}(P) = (x - L, -y + 2L, -z)$; the square of their comoving distance is

$$d^2 = 5L^2 - 8Ly + 4y^2 + 4z^2 \ . \qquad (3)$$

LeLaLu's method is to make a table of the comoving distances between all pairs of images in a given catalog, then plot $n(d)$, the number of occurrences of each value of d, versus $(d/L)^2$. As suggested by Eq. (3), such plots should have peaks on integral numbers, which will depend on the particular topology and the motions separating the images. Real catalogs are not yet deep enough to reveal these peaks, especially if L is very large. But LeLaLu do several simulations, for the six CEMs, with $L = 1500h^{-1}$Mpc, a pseudo-random distribution of 50 sources in the FP, and a catalog depth of $Z = 4$. We might think of these sources as galaxy clusters or superclusters, in a "top-down" picture of structure formation, or as

galaxies and protogalaxies in the now favored "bottom-up" picture - cf. Peebles [13].

We have done other simulations, with our present intention of showing an effect of a nontrivial space topology on a *large* universe. Our sources include our Galaxy at the observer's position (0,0,0), the other ones being at pseudo-randomly chosen positions in the FP. (Strictly, except in case E1, which is homogeneous in the geometric sense, putting the observer at the FP's center is un-Copernican; but it does not make much difference in the present study.) And we supposed images distributed in all of the observable universe (depth $= R_H$, hence highly unrealistic at present; but not unimaginable, given the progress in such powerful observational tools as gravitational lensing and the Lyman alpha forest of quasar absorption lines), presuming that the precursors of today's structures have been present, at least in embryonic form, from recombination time onwards; see also Fagundes [14]. Then we calculated and plotted $n(d)$ for both $L = 0.7R_H$ (20 sources) and $L = 1.2R_H$ (40 to 101 sources). For $L = 4200h^{-1}$ Mpc our plots for $E1 - E3$ agree with LaLeLu's in their common range. For $E5$ and $E6$, we got plots different from theirs, because we chose as our FP a hexagonal prism with different dimensions: in our case the hexagon's shortest diameter, not its side, is equal to L, the length of the vertical edges.

Fig. 1 shows our both plots for the corrected $E4$ universe and for our version of the $E5$ case. There are four neat spikes on integral numbers for the smaller universes, and one for the larger $E5$. Model $E4$ does not show a peak at $d = L$, because the effect of a translation L in each generator is strongly masked by the accompanying rotations - cf. Eqs. (2). The larger $E4$ does not show a significant peak at $d = L\sqrt{2}$ either; this is almost twice the horizon's radius, and few ocurrences of this distance are expected. In this case the plot looks like one for an ordinary (infinite) EdS model, and we would have to look for other indicators of a nontrivial topology - see below.

We have not made simulations for the asymmetric T^3 models of Ref. [8]. But it should be clear that, for example, in their model T^1 with $L_1 = 3000h^{-1}$ Mpc there will be many pairs separated by distances L_1 and $2L_1$. The reader can convince herself or himself of this through a simple sketch of five cells in a row.

III. FINAL REMARKS

It is true that large ($L/R_H \sim 1$) closed models do not solve the homogeneity problem. We may still have inflation, as admitted by E&S even for their small models. Actually, most research on cosmic global topology has been unconcerned with explaining homogeneity - see LaLu for a review; two recent examples are Ref. [5], where the limit on L was obtained from a consideration of inflation theory, and Jing and Fang [15], who find $L \approx 0.8R_H$ for an $E1$ universe as an explanation for a possible infrared cutoff in quantum field theory.

The predictions of cosmic crystallography may eventually become testable. Other recent suggestions for verifying multiple connectedness are Cornish, Spergel, and Starkman's [16] "circles in the sky," and Roukema's [9] probabilities for finding repeated images of groups of quasars. On the theoretical side, the compactness of space is called for by quantum cosmology; cf. Hartle and Hawking [17], or Zeldovich and Starobinsky [18]. So, even if CEM universes cannot be small (which is by no means certain; see Introduction), it makes very much sense to continue to explore their cosmological possibilities and, more generally, those of the rich class of compact 3-manifolds with a locally homogeneous geometry - see Refs. [19 – 21], for example.

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[1] J. A. Wolf, *Spaces of Constant Curvature* (McGraw-Hill, New York, 1967)

[2] G. F. R. Ellis, Gen. Relat. Gravit. **2**, 71 (1971)

[3] M. Lachièze-Rey and J.-P. Luminet, Phys. Rep. **254**, 135 (1995); LaLu

[4] G. F .R. Ellis and G. Schreiber, Phys. Lett. A **115**, 97 (1986); E&S

[5] I. Y. Sokolov, JETP Lett. **57**, 617 (1993)

[6] D. Stevens, D. Scott, and J. Silk, Phys. Rev. Lett. **71**, 20 (1993)

[7] A. de Oliveira-Costa and G. F. Smoot, Astrophys. J. **448**, 477 (1995)

[8] A. de Oliveira-Costa, G. F. Smoot, and A. A. Starobinsky, Astrophys. J. **468**, 457 (1996)

[9] B. F. Roukema, Mon. Not. Roy. Ast. Soc. **283**, 1147 (1996)

[10] R. Lehoucq, M. Lachièze-Rey, and J.-P. Luminet, Astron. and Astrophys. **313**, 339 (1996); LeLaLu

[11] A. Bernui, G. Gomero, M. J. Rebouças, and A. F. F. Teixeira, report CBPF-NF-027/97 of Centro Brasileiro de Pesquisas Físicas

[12] G. I. Gomero, personal communication (1996)

[13] P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton University, Princeton, 1993), Chapter 15

[14] H. V. Fagundes, Astrophys. J. **470**, 43 (1996)

[15] Y.-P. Jing and L.-Z. Fang, Phys. Rev. Lett. **73**, 1882 (1994)

[16] N. J. Cornish, D. N. Spergel, and G. D. Starkman, preprint gr-qc/9602039

[17] J. B. Hartle and S.W. Hawking, Phys. Rev. D **28**, 2960 (1983)

[18] Y. B. Zeldovich and A. A. Starobinsky, Sov. Astron. Lett. **10**, 135 (1984)

[19] H. V. Fagundes, Phys. Rev. Lett. **54**, 1200 (1985); Gen. Relat. Gravit. **24**, 199 (1992)

[20] T. Koike, M. Tanimoto, and A. Hosoya, J. Math. Phys. **35,** 4855 (1994); J. Math. Phys. **38**, 350 (1997)

[21] J. R. Weeks, *The Shape of Space* (Marcel Dekker, New York, 1985)

Figure legend:

FIG. 1. Distribution of comoving distances between images in simulations with 20 (smaller L) or 101 sources. The number of images is about the same $(246 - 280)$ in each case. The bins have width 0.01.

Universe E4
L = 4200/h Mpc

Universe E4
L = 7200/h Mpc

Universe E5
L = 4200/h Mpc

Universe E5
L = 7200/h Mpc